Theoretical Research in Geometry and Algebra and Their Potential Applications in Material Science

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Eriko SHINKAWA (新川 恵理子)

Overview of My Research

Algebra

Mathematics

Geometry

Representation theory

・Virasoro algebra and Schur function

Background

- Virasoro algebra is spanned by the generators $\{L_k | k \in \mathbb{Z}\} \cup \{C\}$, where

$$L_k = \frac{1}{4}\sum_{j\in\mathbb{Z}_{odd}} :a_{-j}a_{j+2k}: + \frac{1}{16}\delta_{k0} id,$$

$$[L_m, L_n] = (m-n)L_{m+n} + {}^{c(m^3-m)\delta_{m+n,0}}/_{12}, \qquad [C, L_n] = 0 \quad (\forall n, m)$$

- Schur's Q function

$$Q_\lambda(t) = \sum_\rho 2^{\frac{l(\lambda)-l(\rho)+\varepsilon}{2}} \zeta_\rho^\lambda \frac{t_1^{m_1} t_3^{m_3} \cdots}{m_1! m_3! \cdots} \quad t = (t_1, t_3, \cdots)$$

where λ is strict partition and $\rho = (1^{m_1}\ 3^{m_3} \cdots)$ is odd partition.

- Connecting other fields

Integrable systems ⟷ **Geometry of Moduli spaces of stable curves**

Kontsevich-Witten theorem

$$P_{01}P_{23} - P_{02}P_{13} + P_{03}P_{12} = 0$$

It is equivalent to the **Virasoro** constraint for the Kontsevich-Witten τ function F. $L_n \exp(F) = 0\ (n \geq -1)$,

Schur functions naturally arise in this context as a suitable representation for the τ-function. For KdV(KP), Schur's Q function (Schur function) is appropriate.

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・**Virasoro algebra and Schur function**

Results

- A formula for Schur Q-functions is presented which describes the action of the Virasoro operators.
- This formula follows from the Plücker-like bilinear identity of Q-functions as Pfaffians.

Theorem 1([Aokage, S, Yamada, '20]) $\lambda = (\lambda_1, \cdots, \lambda_{2m})$ is a strict partition

1. $\sum_{i=1}^{2m} (-1)^i \partial_1 Q_{\lambda_1 \lambda_i} \partial_1 Q_{\lambda_2 \cdots \widehat{\lambda_i} \cdots \lambda_{2m}} = 0$
2. $\sum_{i=1}^{2m} (-1)^i \{\partial_1 Q_{\lambda_1 \lambda_i} \partial_3 Q_{\lambda_2 \cdots \widehat{\lambda_i} \cdots \lambda_{2m}} + \partial_3 Q_{\lambda_1 \lambda_i} \partial_1 Q_{\lambda_2 \cdots \widehat{\lambda_i} \cdots \lambda_{2m}}\} = 0$

Theorem2 ([ASY '21]) For $\lambda = (\lambda_1, \cdots, \lambda_{2m})$ is a strict partition and $k \geq 1$,

$$L_k Q_\lambda = \sum_{i=1}^{2m} (\lambda_i - k)\, Q_{\lambda - 2k\varepsilon_i}, \qquad \lambda - 2k\varepsilon_i = (\lambda_1, \cdots, \lambda_i - 2k, \cdots, \lambda_{2m})$$

Plücker like relation (Schur's Q function)

$$\partial_1 Q_{01} \partial_1 Q_{23} - \partial_1 Q_{02} \partial_1 Q_{13} + \partial_1 Q_{03} \partial_1 Q_{12} = 0$$

Cf. Plücker relation (Schur function)

$$P_{01} P_{23} - P_{02} P_{13} + P_{03} P_{12} = 0$$

Academic Values

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Kontsevich-Witten theorem

Schur function

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Overview of My Research / Future Perspective

Mathematics

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Geometry

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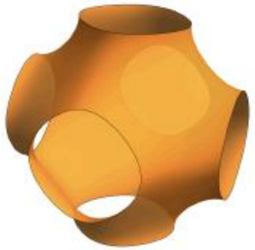
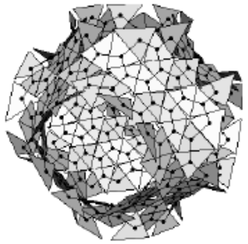
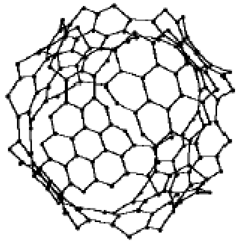
Riemann Geometry

- Surfaces with Anisotropy

Discrete Differential Geometry







Material Science/Process Science

Kneading

Small nanoparticles and their structure

Othere field