

Geometry of surfaces & cluster algebras

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Cluster algebra ... a common combinatorial structure
appearing in various kinds of mathematics.

It is defined for a quiver = oriented graph.
by using mutations.

Integrable Systems

dimer model

Painlevé equation

↓ discretization

q-Painlevé equation

Theoretical Physics

BPS state of 4d $N=2$ SUSY

UV-IR corresp.

S-duality, T-duality (mirror sym)

AGT corresp. spectral network

scattering amplitude

WKB analysis

Stokes phenomena
spectral cover

Representation Theory

rep. of fin. dim. alg.

Kac-Moody Lie alg.

Weyl group.

Cluster Algebra

cluster variety

cluster variable

cluster modular group

↕
mutation loop

Knot theory

polynomial invariants

skein algebras

volume conjecture

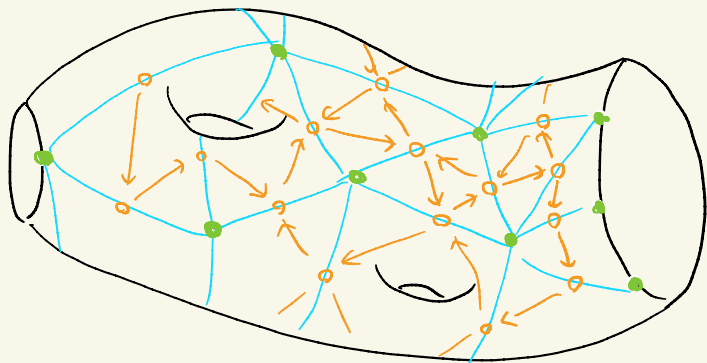
Geometry of Surfaces

(higher) Teichmüller space

quadratic differential

mapping class group

How appear cluster combinatorics in the geometry of surfaces

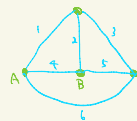
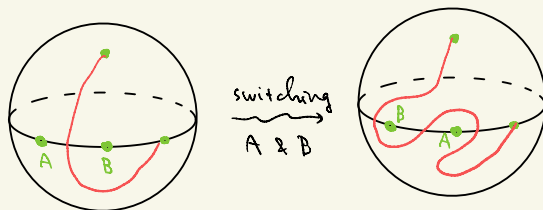
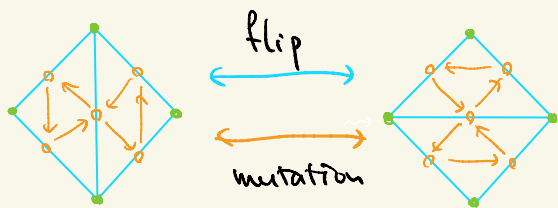


— : a surface

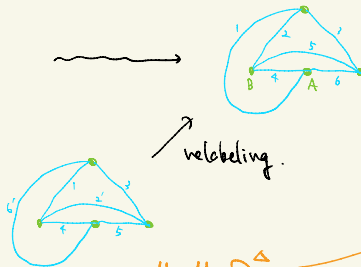
• : punctures

— : an ideal triangulation $\triangle$

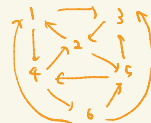
— : corresponding quiver. Q^Δ



flipping
along 2 & 6



relabeling.



$$Q^{\Phi(A)} = Q^\Delta$$

mutation at 2 & 6 $\rightarrow \mu_6 \mu_2 Q^\Delta \xrightarrow{\text{relabeling}}$

