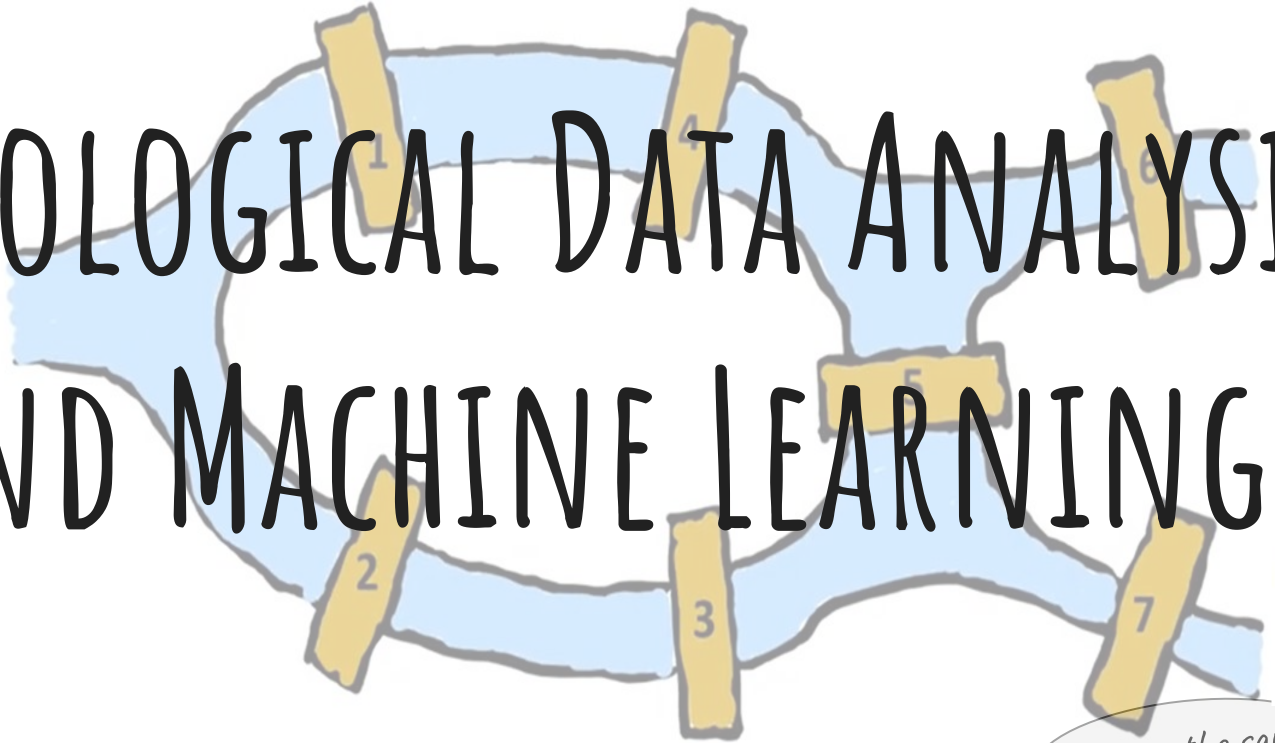
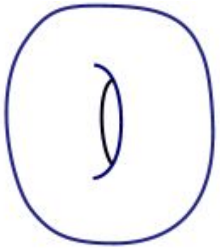
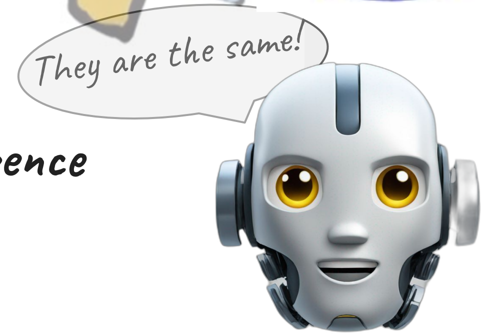
# Topological Data Analysis and Machine Learning

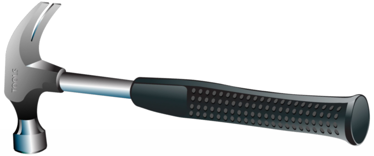




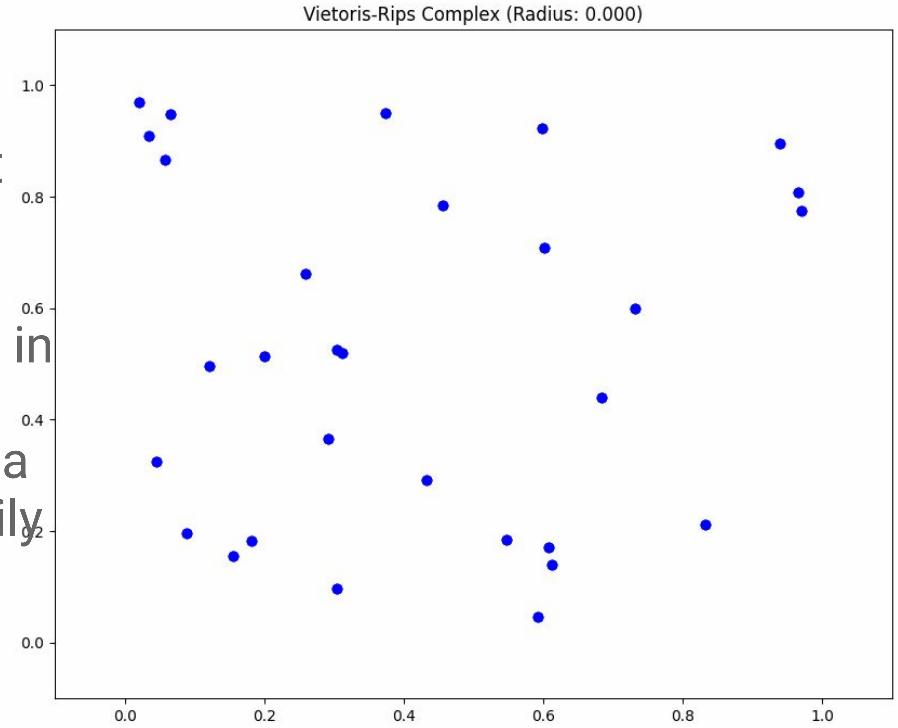

*Bridging Mathematical Insights with Computational Intelligence*

# Topological Data Analysis



Assumption: Data has shape

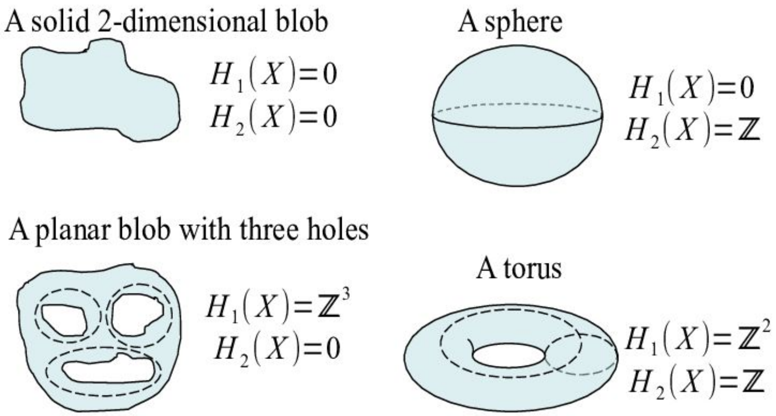
- input is assumed to be a finite set of points coming with a notion of distance between them
- shape is built on the top of the data in order to highlight the underlying topology or geometry. This is often a simplicial complex or a nested family of simplicial complexes, called a filtration

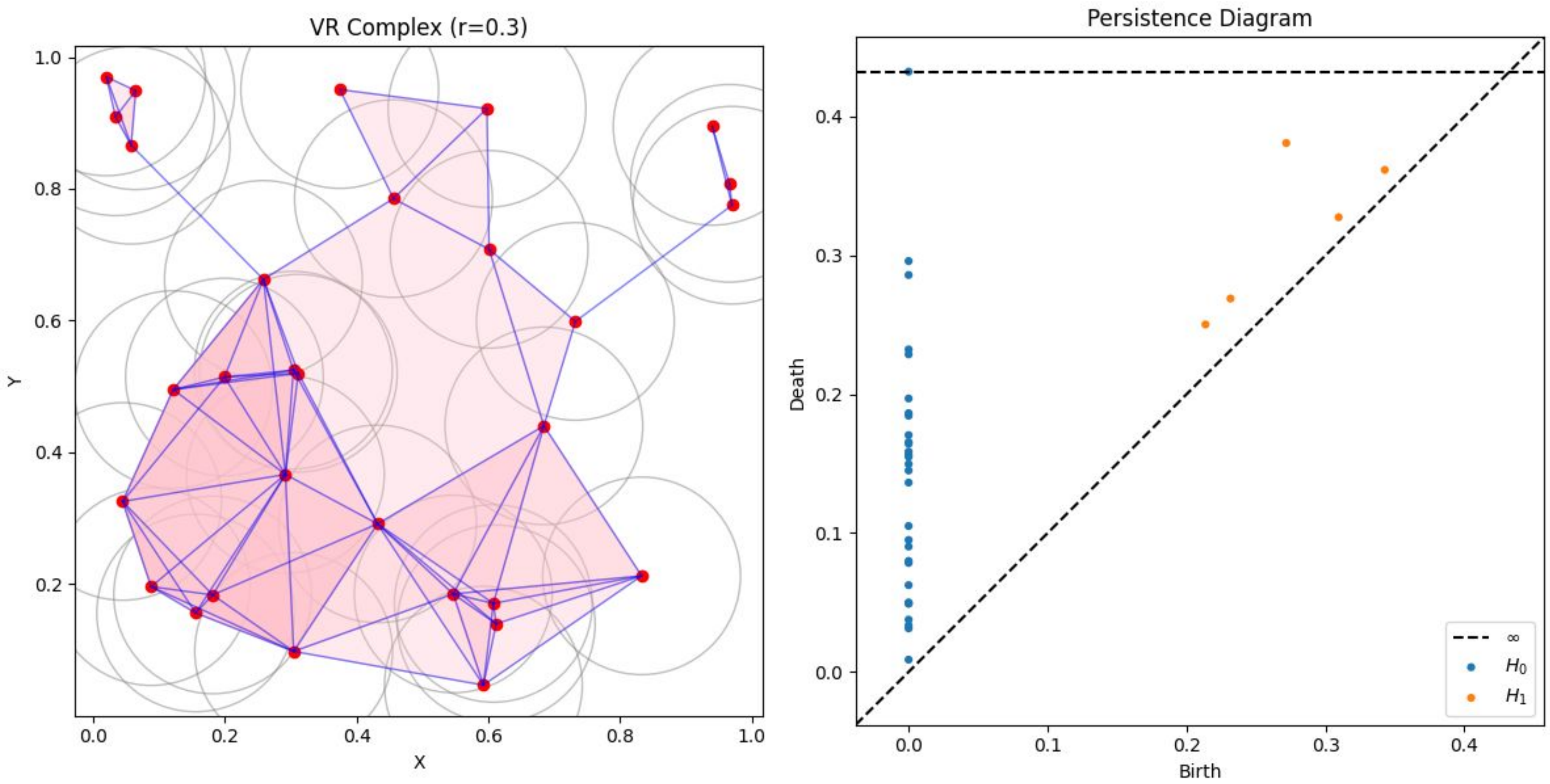


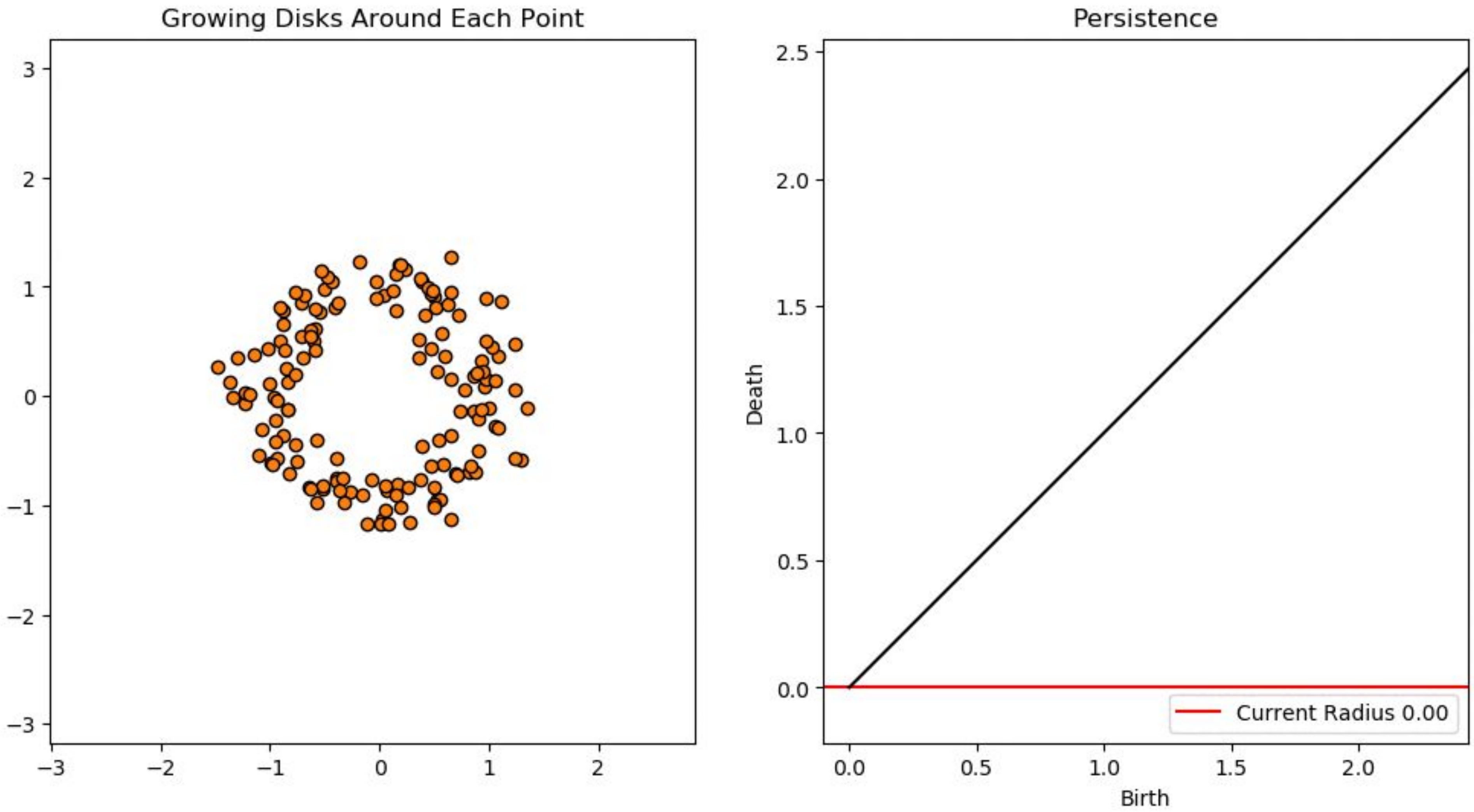
# Persistent Homology

Persistent homology tracks how topological features (like connected components, loops, and cavities) are born, evolve, and die as you progressively change the resolution or scale of your data.

We observe how homology groups change as the complex grows, then we represent results using persistence diagrams or barcodes that show when topological features are born and die.




VR Complex (r=0.3)
Y
X
0.0
0.2
0.4
0.6
0.8
1.0
Persistence Diagram
Death
Birth
0.0
0.1
0.2
0.3
0.4
∞
$H_0$
$H_1$



Growing Disks Around Each Point
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
Persistence
2.5
2.0
1.5
1.0
0.5
0.0
Death
0.0
0.5
1.0
1.5
2.0
Birth
Current Radius 0.00

# Machine Learning (sp. Neural Networks)

Neural networks are artificial intelligence systems that:

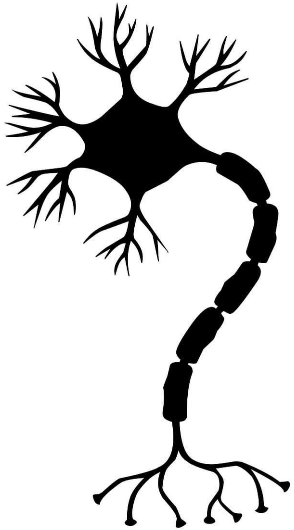
- Mimic biological neural networks
- Process information through interconnected nodes (neurons)
- Learn by adjusting internal connections based on data

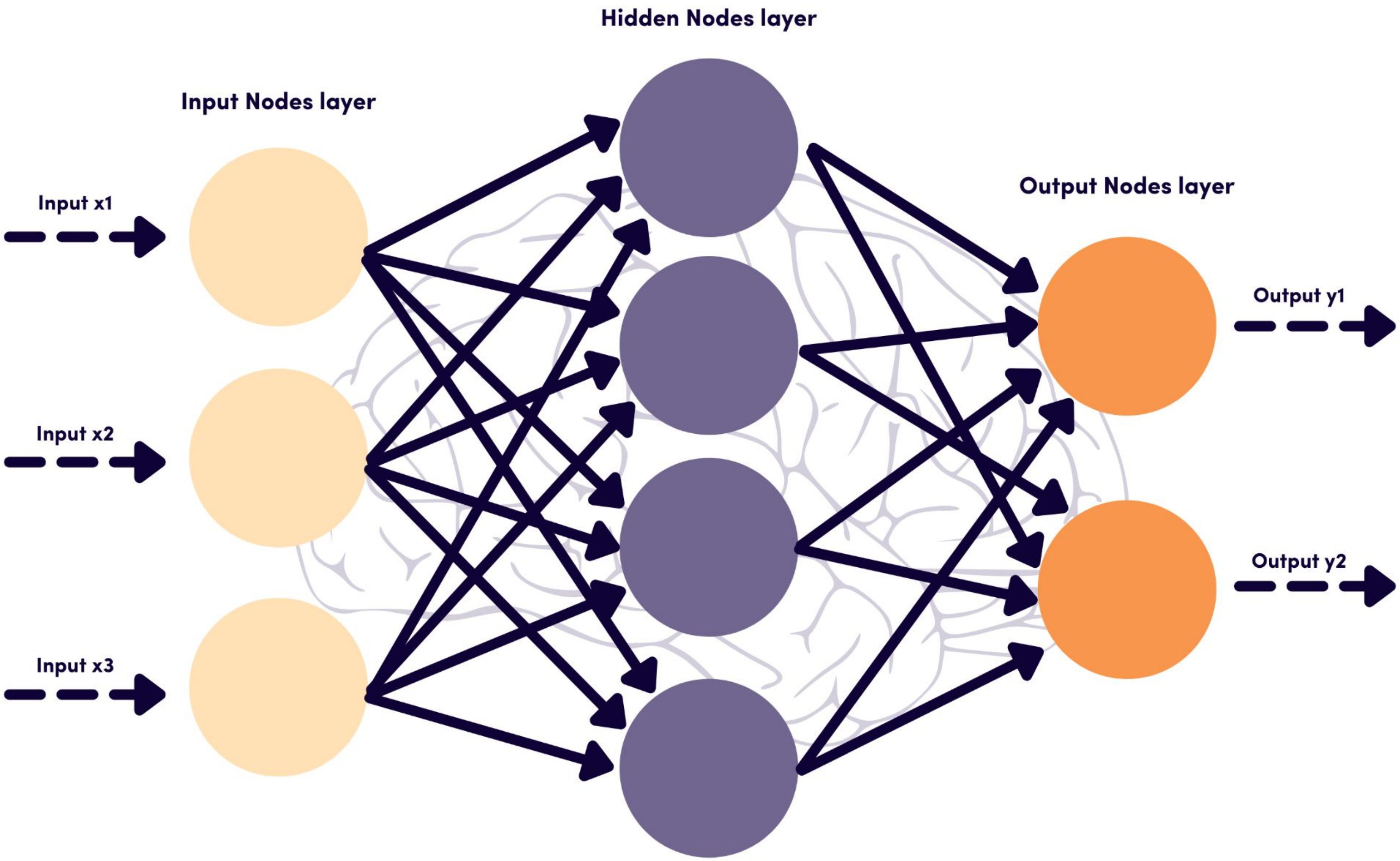
Key Characteristics

- Layered Structure: Consists of input, hidden, and output layers
- Adaptive Learning: Improves performance through continuous training
- Pattern Recognition: Identifies complex relationships in data

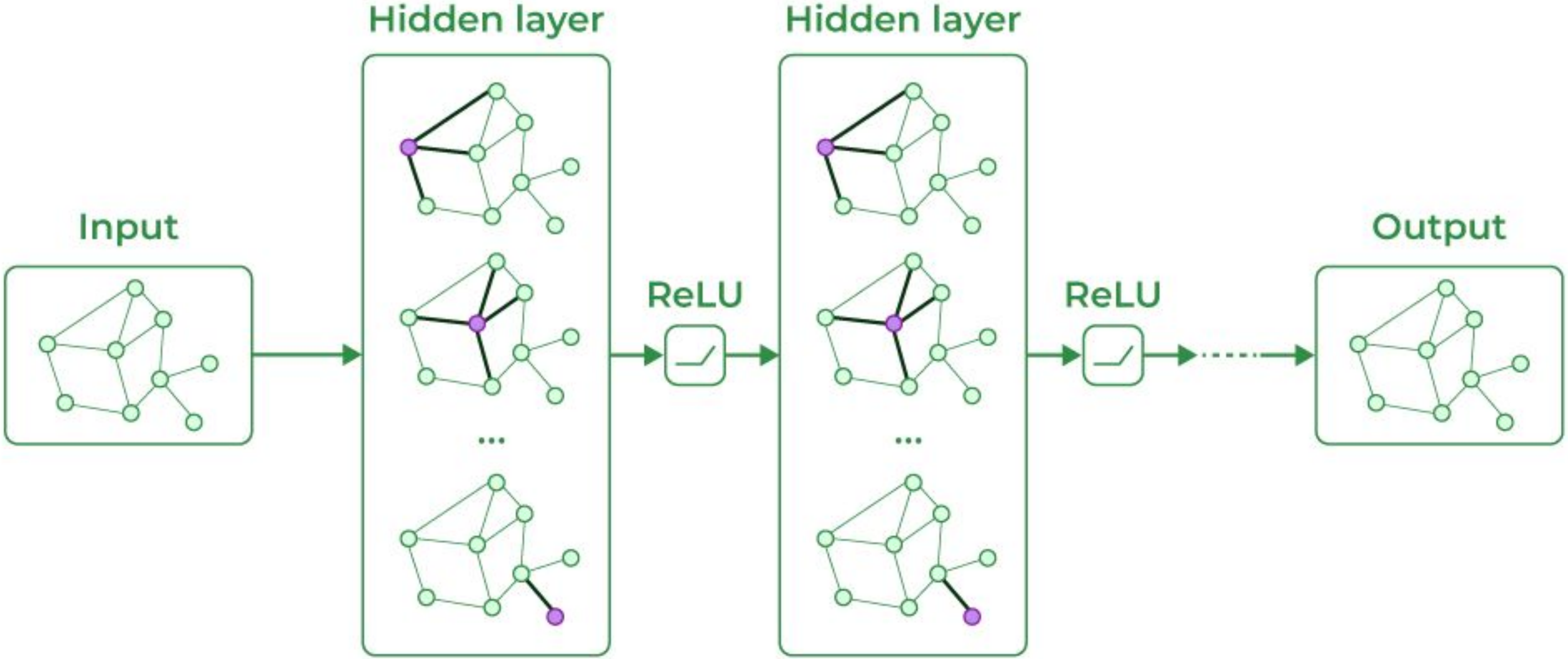
How They Work

1. Input Processing: Receive data through an input layer
2. Hidden Layer Transformation: Perform computational transformations
3. Output Generation: Produce predictions or classifications
4. Continuous Refinement: Adjust weights and biases to improve accuracy




Hidden Nodes layer
Input Nodes layer
Output Nodes layer
Input x1
Input x2
Input x3
Output y1
Output y2

# Sp. sp. Graph Neural Networks



# Technology Integration

### Topological Feature Injection in Graph Neural Networks (GNNs)

Topological Feature Injection involves incorporating persistent homology features into graph convolution layers to enhance the network's ability to capture multi-scale structural information. This approach leverages the topological properties of the graph to improve the learning process.

### Graph Pooling with Persistent Homology

Graph Pooling (GP) is a crucial operation in graph learning methods, which hierarchically aggregates an upper-level graph into a more compact lower-level graph. Aligning persistent homology with graph pooling operations ensures that message passing in the coarsened graph follows persistent sub-topology, leading to improved performance.

### Graphcode: Learning from Multiparameter Persistent Homology

Graphcodes are a novel multi-scale summary of the topological properties of a dataset based on persistent homology. They handle datasets filtered along two real-valued scale parameters, offering an efficient and interpretable summary.