# g-RIPS 2022 Mitsubishi Project B Final Presentation

Justin Lien, Ariana Brown, Lee Fisher, Fumiya Okazaki

Multi-Objective Optimization for Best Early Prediction of Extreme Weather Events

**Academic Mentor**: Dr. Hiroyasu Ando
**Industry Mentors**: Dr. Yusuke Ito and Dr. Hidenobu Tsuji

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## Motivation for the Project

- Early prediction of extreme weather phenomenon is crucial.

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$$\text{Water vapor and wind} \Rightarrow \text{Cumulonimbus clouds}$$
$$\Rightarrow \text{Heavy rain} \Rightarrow \text{Extreme weather event}$$

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- The ability to sense the distribution of water vapor and wind provides larger lead time in detection.
- For this project, we will consider the best placement of high-grade and low-grade LiDAR instruments in Japan to help sense extreme weather.

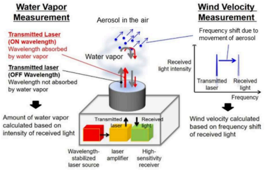
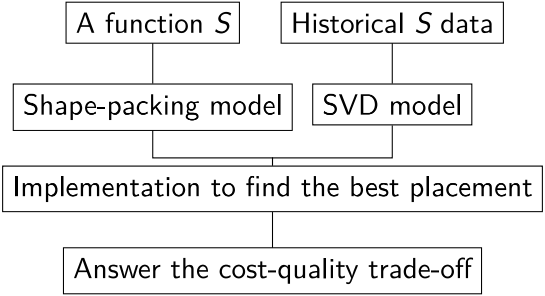



Fig. 1. Measurement principle using LiDAR[3].

Figure: Cumulonimbus Clouds and LiDAR Instruments

**GOAL:** To solve a multi-objective optimization problem regarding the performance of placement and cost of the instruments.
**Strategy:** Depending on the type of data for significance function $S$, to accomplish this we will:



## Definition

The **prediction area** $\Omega_0$ is a bounded finite, real, 2 dimensional manifold (the topography of a city). The **prediction space** $\Omega = \Omega_0 \times (0, \alpha)$ where $\alpha > 0$ can be infinite. This is understood to be the surface of a city and the air space of interest above the surface of the ground.

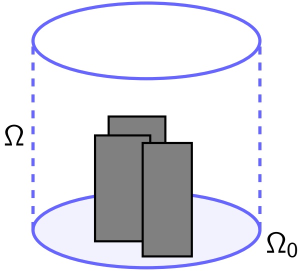



Figure: The prediction space and the prediction area.

## Definition (Water and Wind Significance)

The functions $S_w, S_v : \Omega \times [t_0, t_1] \to \mathbb{R}^+$, where $[t_0, t_1]$ is an interval of time where an extreme weather event would occur.

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$$S(\mathbf{x}, z, t) = S_w(\mathbf{x}, z, t) + S_v(\mathbf{x}, z, t) \tag{1}$$

The sum is simply **significance function**.

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- There are two types of LiDAR instruments: low-grade and high-grade.

$$\text{Low-grade} \Rightarrow \text{Cost \& Viewing set} \searrow$$
$$\text{High-grade} \Rightarrow \text{Cost \& Viewing set} \nearrow$$

- Assuming uniform continuity of the significance function, let the cheaper instrument be a thin vertical cylinder where the reading is constant for each height section with controllable error.

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- We will let the viewing set of the expensive instrument to be a wider conical set with viewing angle $\theta$.

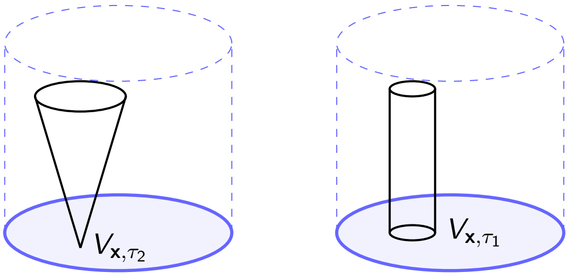


Figure: Viewing sets for high and low quality instruments.

## Definition

A **configuration** $\chi_n$ of $n$ instruments is a set of $n$ pairs, $(\mathbf{x}_1, \tau_1), \ldots (\mathbf{x}_n, \tau_n)$ that represent the placements and types of instruments. The viewing set of $\chi_n$ is the union of the viewing sets of each of the instruments.

$$V_{\chi_n} = \bigcup_{i=1}^{n} V_{\mathbf{x}_i, \tau_i} \quad (2)$$

$$Q(\chi_n) = \int_{t_0}^{t_1} \int_{V_{\chi_n}} S(\mathbf{x}, z, t) d\mathbf{x} dz dt \quad (3)$$

$$C(\chi_n) = \sum_{i=1}^{n} C(\tau_i) \quad (4)$$

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3. **Find optimizers:** We will select a Pareto efficient compromise between cost and quality.

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$$S = p(\mathbf{x})g(z)f(t) \tag{5}$$

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- $g(z)$: meteorological information about what altitudes are important to observe for predicting an extreme rainfall event.
- $f(t)$: description of diurnal and seasonal variation.

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$$V_{\mathbf{x},\tau_2} \approx B(\mathbf{x}, r_2) \times \mathbb{R}^+ \quad (6)$$

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where $r_i$ is the radius of the circular section in each cylinder.

- From (6) we can simplify the viewing set as follows:

$$V_{\chi_n} = \bigcup_{i=1}^{n} V_{\mathbf{x}_i,\tau_i} \approx \tilde{V}_{\chi_n} = \mathbb{R}^+ \times \left( \bigcup_{i=1}^{n} B(\mathbf{x}_i, r_i) \right)$$

$$\tilde{Q}(\chi_n) = \int_{t_0}^{t_1} \int_{\cup_i B(\mathbf{x}_i, r_i)} \int_{\mathbb{R}^+} p(\mathbf{x}) g(z) f(t) dz d\mathbf{x} dt = \tilde{K} \int_{\cup_i B(\mathbf{x}_i, r_i)} p(\mathbf{x}) d\mathbf{x}.$$

- Optimizing $\tilde{Q}(\chi_n)$ can be understood as a circle packing problem.

$$\hat{Q}(\mathbf{x}_1, \ldots \mathbf{x}_n, r_1, \ldots r_n) = \int_{\cup_i B(\mathbf{x}_i, r_i)} p(\mathbf{x}) d\mathbf{x} \tag{8}$$

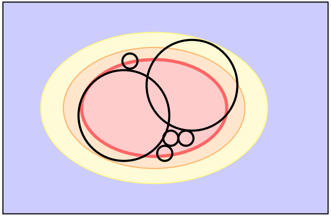

Figure: An example function $p(\mathbf{x})$ given by a heat map, and a choice of configuration. Redder colors correspond to higher significance.

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$$\text{Place larger circles one by one} \Rightarrow \text{Place smaller circles one by one}$$

- In our experiments, we found

$$\text{Greedy} > \text{Biased} > \text{Uniform} > \text{Gradient Descent}$$

- Later, we will implement the greedy algorithm to place the sensors.

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- Most of the same algorithms and strategies can be adapted from the 2D to the 3D case.
- The main principles and code for the 2D and 3D versions are similar.

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In the next example, we will use a 2D significance function (based on historical precipitation) to optimally place the sensors in Tokyo.

## A significance function for Tokyo

- We used rainfall data from JMA[1] to create $p(x)$.

[1] Japan Meteorological Agency

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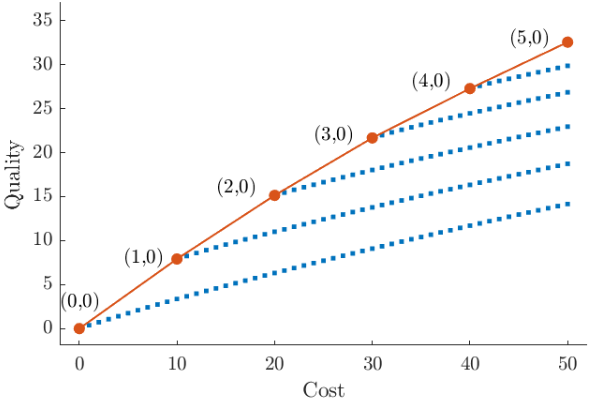
- Don't place sensors over the ocean!
- We used the Greedy Algorithm to place 14 sensors around Tokyo.

[1] Japan Meteorological Agency

# Placing Sensors in Tokyo


Figure: Four high quality sensors and ten low quality sensors are placed around Tokyo. The significance function is given as a heat map.

## Pareto Optimality



## About Significance Function

- $S(\mathbf{x}, z, t)$ predicts an extreme weather event before the formation of the Cumulonimbus.
- It can be approximated by some function of water vapor and wind field[2]

$$S(\mathbf{x}, z, t) = F(v(\mathbf{x}, z, t), \vec{w}(\mathbf{x}, z, t))$$

- The sensors sample $v$ and $\vec{w}$, which is equivalent to sampling $S$.

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There are two problems:

- We do not know the atmospheric science to describe exactly $F$.
- We do not have access to good data sets of $v$ and $\vec{w}$.

[2] Actually, it also depends on the chemical components, size distribution of aerosols, over-saturation condition, etc.

- Construct $S(\mathbf{x}, z, t)$ with the correct data:
  - Some random and some periodic features.
  - Smooth dependence over space and time.
- Using historical data, we want to find optimal placements of sensors; then we use readings from those sensors to interpolate the significance function over a large area.

- We will study the variable “temperature” instead of “significance”.
  - Gathering “temperature” data is straightforward.
  - Temperature and significance both have random and periodic features.
  - Temperature and significance are both real valued functions of time and space.
- We will place a few point sensors and use them to read the “temperature function” for all of Japan.
- If a historical significance dataset is given, then the same algorithm can be used to place the sensors.

- Let $\Omega$ be a mesh of our domain with $N$ elements.
- Let $X$ be our data matrix with size $N \times T$.
  - $T$ is the number of time samples, we used hourly data in our case.
  - The columns of $X$ are the temperatures of all of Japan at each hour over the span of a year.

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$$X = U \Sigma V^T$$

  - $\Sigma$ is a square diagonal matrix of with real entries.
  - $U$ and $V$ have orthonormal columns.
  - Columns of $U$ corresponding to small entries in $\Sigma$ are less important.

- $X_t$ is the temperature field of Japan at time $t$, an $N \times 1$ vector. Let $r$ be the number of modes and $x_r$[3], called strength, be such that,

$$X_t = U_r x_r + \nu_r$$

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- Place point sensors at sites $i_1, \ldots i_p$. The temperature reading is

$$y_t = \begin{bmatrix} 0 & \cdots & 1_{(1,i_1)} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 1_{(p,i_p)} & \cdots & 0 \end{bmatrix} (U_r x_r + \nu_r)$$
$$= HU_r x_r + H\nu_r = Cx_r + H\nu_r$$

- $\nu_r$ depends on smaller singular values, but we treat it as random noise.

[3] It depends on $t$.

- We reconstruct the temperature field by the equation

$$\tilde{X}_t = U_r C^{-1} y_t$$

- When $C$ is not square, the inverse is in the least squares sense.
- The error in the approximation is:

$$\tilde{X}_t - X_t = U_r C^{-1} H \nu_r$$

- The size of the error vector will depend on $C^{-1}$.
- If the determinant is large, then the error will be amplified.

- To optimize our placement, we should consider[4]

$$\max det(C) \text{ or } \min det(C^{-1})$$

- Placing sensors at sites $i_1, \ldots i_p$ means that the rows of $C$ will be rows $i_1, \ldots i_p$ of $U_r$.
- We use the Greedy algorithm and Schur-complement to selects rows of $U_r$ which increase $det(C)$ as much as possible.

[4] $C = (C^{-1})^{-1}$

## Temperature matrix for Japan

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- This strategy is used to read temperature, but it could also be used to read the significance function.
- On the scale of the entire Japan there is **NOT MUCH** difference between LGS and HGS.
- We used Dr. Nonomura's approach to place 10 point sensors (LGS) around Japan.

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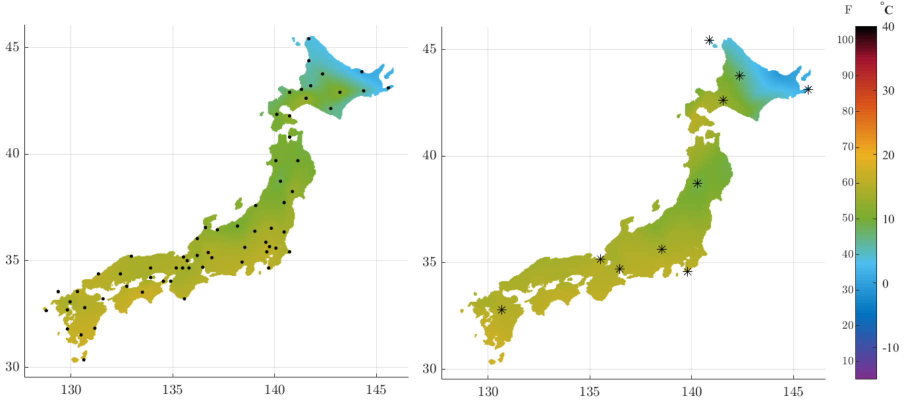


Figure: 10 point sensors infer the temperature on March 15 2021.

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- On the Tokyo city scale there is a **LARGE** difference between high and low grade sensors.
- The high grade sensors can see many locations, but the temperature profiles of adjacent sites are nearly linearly dependent.
- We had to modify the method in order to use the HGS.

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- We use the singular values of the HGS's viewing set to judge the sensors "essential dimension".
- If a HGS has "essential dimension" $n$, then $n$ LGSs would give a measurement of equal quality.

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  - depends on placement; it is higher in areas with localized variation.
- In our experiments, HGSs usually had essential dimension 2 or 3, but this is because our data was too coarsely sampled.

## Looking at Tokyo Again

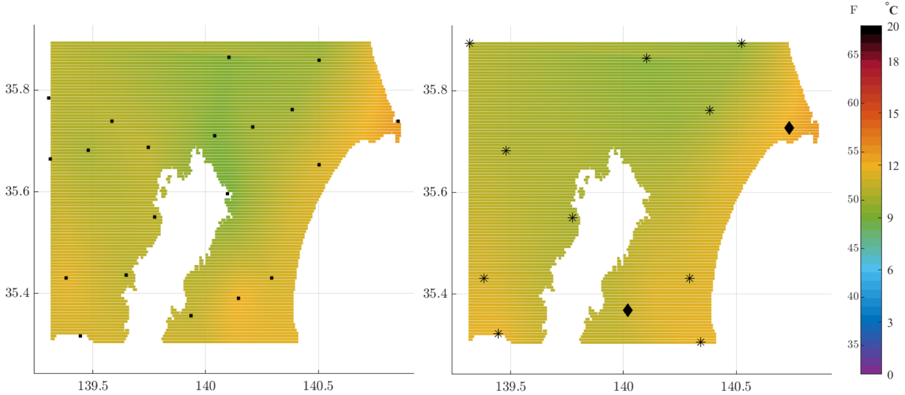


Figure: Two high grade sensors and eight low grade sensors placed around Tokyo on January 11 2021.

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- Any method to interpret the sensors based on historical data will slowly degrade in quality because of climate change.

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- Any method to interpret the sensors based on historical data will slowly degrade in quality because of climate change.
- For our SVD methods we ignored the fact that HGS can see some of the ocean even if it is on the land, this can be remedied but we didn't have the time to implement it.

## Conclusion

- To solve the multi-objective optimization problem, we offer the circle packing approach and Dr. Nonomura's approach to optimally place the sensors.

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| Tokyo | |
|---|---|
| Shape-Packing | More high-grade sensors |
| SVD | Data dependent |

| Japan | |
|---|---|
| SVD | Low-grade sensors |

## Reference


- Syo Yoshida. Study on Short-Term Forecast of Localized Heavy Rainfall Based on the Statistical Characteristics of Cumulonimbus Clouds Observed by Ka-band Doppler Radars. PhD thesis, University of Tsukuba,2021 .
- Masaharu Imaki, Kenichi Hirosawa, Takayuki Yanagisawa, Shumpei Kameyama, and Hiroaki Kuze. Wavelength selection and measurement error theoretical analysis on ground-based coherent differential absorption lidar using 1.53-$\mu m$ m wavelength for simultaneous vertical profiling of water vapor density and wind speed: publisher's note. Appl. Opt., 59(8): 2667–2667, Mar 2020.

- Masaharu Imaki, Hisamichi Tanaka, Kenichi Hirosawa, Takayuki Yanagisawa, and Shumpei Kameyama.Demonstration of the 1.53- m coherent dial for simultaneous profiling of water vapor density and wind speed. Opt. Express, 28(18): 27078–27096, Aug 2020.
- G.W. Petty. A First Course in Atmospheric Thermodynamics. Sundog Pub., 2008.
- Kelly. Continental and local winds.
- Keigo Yamada, Kumi Nakai, Takayuki Nagata, Keisuke Asai, Yasuo Sasaki, Yuji Saito, Taku Nonomura and Daisuke Tsubakino. Determinant-based fast greedy sensor selection algorithm.